[8] in a small radius burner, which may prove significant because of retardation of the flame in the gas suspension as compared to the case of a pure gas mixture. Radiant heat loss from radiating particles may also play some role.

Consideration of these effects requires further development of the model of powder—flame interaction. The theory presented here indicates that all the basic experimental facts presented above are adequately described by a model including only the thermal interaction mechanism.

NOTATION

T_g, c_g, ρ_g, λ_g, gas temperature, specific heat, density, and thermal conductivity; T_p, c_p, ρ_p, w_p, s_p, particle temperature, specific heat, density, volume, and area; E, k, R, Q, activation energy, preexponential term, ideal gas constant, thermal effect of combustion of initial reagent b; θ_p, θ_g, dimensionless gas and particle temperatures; $\tau = t/t_+$ and $\xi = x/x_+$, dimensionless time and coordinate; ω, B, $\varkappa$, $l = L/x_+$, τ_1, parameters; L, half width of ignition hearth; l_*, critical hearth size.

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METHODS OF RESEARCHING THERMOPHYSICAL PARAMETERS AND PHENOMENA BY MEANS OF NONSTATIONARY-FREQUENCY MEASUREMENTS.

PART 2. STEP AND INSTANTANEOUS HEATING METHODS

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Various types of instantaneous and stepped heat source are considered, which act in unbounded bodies. A method has been devised for using the solutions to define the thermophysical parameters by means of nonstationary-frequency measurement methods.

Pulse, stepped, and periodic heating methods are [1] the most promising and correspond to current requirements as regards speed, accuracy, and informativeness. Phase and frequency measurements may be made instead of amplitude ones to considerable advantage as regards resolution and speed [1], but in that study, the restricted volume meant that it dealt with only one form of step methods, namely a semiinfinite body with boundary conditions of the first kind. That however demonstrated the main advantages of the formulation and solutions. Therefore, here and subsequently we avoid giving excess details.

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The most general case is where the thermal pulse length τ_0 takes any finite value, which will be considered in the next part of this study, while here we restrict consideration to two asymptotic cases: steps, $\tau_0 \to \infty$, and instantaneous sources, $\tau_0 \to 0$. We consider boundary conditions of a second kind as those most widely used in such measurements, together with three basic heat source types: point, line, and planar acting in an unbounded body.

The source—function method represents the most natural approach here [2, 3].

The function

$$T(x,\ y,\ z,\ \tau - t) = \frac{Q_1/c}{[2\sqrt{\pi a(\tau - t)}]^3} \exp\left[-\frac{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}{4a(\tau - t)}\right] \tag{1}$$

represents a fundamental solution to the conduction equation and is called the instantaneous source temperature-influence function; it is the solution for the temperature distribution in an unbounded body at any instant arising from an instantaneous source Q_1 at x_1, y_1, and z_1 at time $\tau = t$.

We integrate (1) over the spatial coordinates to get expressions for a line source

$$T(x,\ y,\ \tau - t) = \frac{Q_2/c}{[2\sqrt{\pi a(\tau - t)}]^2} \exp\left[-\frac{(x-x_1)^2+(y-y_1)^2}{4a(\tau - t)}\right] \tag{2}$$

and a planar one

$$T(x,\ \tau - t) = \frac{Q_3/c}{2\sqrt{\pi a(\tau - t)}} \exp\left[-\frac{(x-x_1)^2}{4a(\tau - t)}\right] \tag{3}$$

We put $t = 0$ to write (1)-(3) as a general expression

$$T_{\delta i} = \alpha_i \tau^{d_i} \exp[-\sigma_i^2/\tau]. \tag{4}$$

Here $i = 1, 2, 3$: 1 point source, 2 line, 3 planar; $\alpha_i = Q_i/c\,(4\pi a)^{d_i}$, $[Q_1] = J$, $[Q_2] = J/m$, $[Q_3] = J/m^2$, $d_1 = -3/2$, $d_2 = -1$, $d_3 = -1/2$, $r_1^2 = (x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2$, $r_2^2 = (x-x_1)^2 + (y-y_1)^2$, $r_3^2 = (x-x_1)^2$, $\sigma_i^2 = r_i^2/4a$.

The rate of change in temperature on instantaneous heating is found by differentiating (4):

$$V_{\delta i} = \alpha_i \tau^{d_i - 1}\left[\frac{\sigma_i^2}{\tau} + d_i\right] \exp\left(-\frac{\sigma_i^2}{\tau}\right). \tag{5}$$

Then (5) for $i = 1$ (point source) is the fundamental solution to the conduction equation written for temperature change rates.

We integrate (5) with $i = 1$ over the spatial coordinates to get expressions for the temperature-change patterns for line and planar sources in (5) ($i = 2$ and 3).

We integrate (1)-(3) with respect to t from 0 to ∞ to get expressions for the temperature distributions in unbounded bodies on step heating that coincide with those derived in [2, 4, 5]:

$$T_{c1} = \alpha_{c1} \frac{\sqrt{\pi}}{\sigma_1} \operatorname{erfc}\left(\frac{\sigma_1}{\sqrt{\tau}}\right), \tag{6}$$

$$T_{c2} = \alpha_{c2}\left[-\operatorname{Ei}\left(-\frac{\sigma_2^2}{\tau}\right)\right] = \alpha_{c2} E_1\left(\frac{\sigma_2^2}{\tau}\right), \tag{7}$$

$$T_{c3} = \alpha_{c3} 2\sqrt{\pi\tau}\, \operatorname{ierfc}\left(\frac{\sigma_3}{\sqrt{\tau}}\right), \tag{8}$$

where $\alpha_{ci} = \frac{q_i}{c}(4\pi a)^{d_i}$, and q_i is the heat flux.

We differentiate (6)-(8) with respect to τ to get expressions for the rate of change in temperature in an unbounded body on step heating:

$$V_{ci} = \alpha_{ci} \tau^{d_i} \exp(-\sigma_i^2/\tau). \tag{9}$$

Formulas (9) and (4) agree apart from a constant factor.

One can also derive (9) if τ in (5) is replaced by $\tau - t$ and the expression is integrated with respect to t from 0 to ∞; we show that these expressions can be used to derive the thermophysical parameters, beginning with step methods.

Let the rates V_1 and V_2 corresponding to τ_1 and τ_2 in step heating be known; we take the ratio and use (9) with the subscripts i and c here and subsequently omitted to get

$$\gamma = \frac{V_2}{V_1}\left(\frac{\tau_2}{\tau_1}\right)^{-d} = \exp \sigma^2 \frac{\tau_2 - \tau_1}{\tau_1 \tau_2},$$

so

$$a = \frac{r^2 \Delta\tau}{4\tau_1\tau_2 \ln \gamma}, \tag{10}$$

where $\Delta\tau = \tau_2 - \tau_1$.

We rewrite (9) as

$$V = \frac{q}{c}(\pi r^2)^d \left(\frac{\sigma^2}{\tau}\right)^{-d} \exp\left(-\frac{\sigma^2}{\tau}\right), \tag{11}$$

and substitute (10) into (11) to get

$$c = \frac{q}{V_1}\left[\frac{\pi r^2 \Delta\tau}{\tau_2 \ln \gamma}\right]^d \exp\left[-\frac{\tau_2 \ln \gamma}{\Delta\tau}\right]. \tag{12}$$

As $\lambda = ac$,

$$\lambda = \frac{q\pi^d}{4V_1\tau_1}\left[\frac{r^2 \Delta\tau}{\tau_2 \ln \gamma}\right]^{d+1} \exp\left[-\frac{\tau_2 \ln \gamma}{\Delta\tau}\right]. \tag{13}$$

For $\Delta\tau \ll \tau_1, \tau_2$ and $\Delta V = V_2 - V_1 \ll V_1, V_2$ (10) simplifies but loses accuracy:

$$a = r^2/4\tau_1\left[\frac{\Delta V}{\Delta\tau}\frac{\tau_1}{V_1} - d\right]. \tag{14}$$

(14) becomes exact for $\Delta\tau \to 0$:

$$a = r^2/4\tau\left[\frac{V'\tau}{V} - d\right]. \tag{15}$$

As the (14) and (15) cases are analogous, we write expressions for c and λ only for (15):

$$c = \frac{q}{V}\left[\frac{\pi r^2}{\eta - d}\right]^d \exp(d - \eta), \tag{16}$$

$$\lambda = \frac{q\pi^d}{4V\tau}\left[\frac{r^2}{\eta - d}\right]^{d+1} \exp(d - \eta), \tag{17}$$

where $\eta = V'\tau/V = V'/\bar{V}'$ is the step parameter for the $V(\tau)$ curve, $\bar{V}' = \frac{1}{\tau}\int_0^\tau V' d\tau = V/\tau$; and $\bar{V}'$ is the mean V' during $[0,\tau]$. The $V(\tau)$ curves have the origin as common point and show two asymptotic forms of behavior: $V = Au(\tau)$ ($V' \to 0$, $\bar{V}' \to \infty$, $\eta \to 0$) and $\tau = Bu(V)$ ($V' \to \infty$, $\bar{V}' \to 0$, $\eta \to \infty$), where A and B are constants and $u(\tau)$ and $u(V)$ are step functions. Parameter η characterizes the closeness of the curve to one of the asymptotes and represents the relation between behavior of the curve near $\tau = t$ and the average behavior of $[0,\tau]$.

If one measures the rates of change in temperature at two different points and records the times τ_1 and τ_2 when $V(r_1, \tau_1) = V(r_2, \tau_2)$, one gets a further accurate and simple method of deriving a, λ, and c:

$$V_1 = \alpha\tau_1^d \exp(-r_1^2/4a\tau_1) = \alpha\tau_2^d \exp(-r_2^2/4a\tau_2) = V_2,$$

$$a = \left(\frac{r_2^2}{\tau_2} - \frac{r_1^2}{\tau_1}\right) \Big/ 4d \ln(\tau_2/\tau_1), \tag{18}$$

$$c = \frac{q}{V_1}\left[\frac{\pi\tau_1\left(\frac{r_2^2}{\tau_2} - \frac{r_1^2}{\tau_1}\right)}{d \ln(\tau_2/\tau_1)}\right]^d \exp\left[\frac{r_1^2 d \ln(\tau_2/\tau_1)}{\tau_1\left(\frac{r_1^2}{\tau_1} - \frac{r_2^2}{\tau_2}\right)}\right], \tag{19}$$

$$\lambda = \frac{q}{V_1} \frac{(\pi\tau_1)^d}{4} \left[\frac{\frac{r_2^2}{\tau_2} - \frac{r_1^2}{\tau_1}}{d \ln(\tau_2/\tau_1)} \right]^{d+1} \exp\left[\frac{r_1^2 d \ln(\tau_2/\tau_1)}{\tau_1 \left(\frac{r_1^2}{\tau_1} - \frac{r_2^2}{\tau_2} \right)} \right]. \quad (20)$$

(9) has its maximum when

$$\tau_{\max} = \sigma^2/(-d). \quad (21)$$

Then

$$a = r^2/4(-d)\,\tau_{\max}, \quad (22)$$

$$c = \frac{q}{V_{\max}} \left(\frac{\pi r^2}{-d} \right)^d \exp d, \quad (23)$$

$$\lambda = \frac{q\pi^d}{4V_{\max}\tau_{\max}} \left(\frac{r^2}{-d} \right)^{d+1} \exp d. \quad (24)$$

It is complicated to use (6)-(8) because they contain special functions, but this becomes possible because we are interested mainly in the initial instants, when the unbounded-body approximation applies closely. Then asymptotic expansions exist for $x \gg 1$ in [2, 6] in (6)-(8):

$$\operatorname{erfc} x \approx \frac{1}{\sqrt{\pi}} \frac{\exp(-x^2)}{x} \left(1 - \frac{1}{2x^2} + \frac{3}{4x^4} - \ldots \right),$$

$$E_1(x) = -\operatorname{Ei}(-x) \approx \frac{\exp(-x)}{x} \left(1 - \frac{1!}{x} + \frac{2!}{x^2} - \ldots \right)$$

and on considering V/T, we get

$$a = r^2/4\tau[\eta - (d+2)], \quad (25)$$

where $\eta = V\tau/T = T'\tau/T$ is the step parameter for $T(\tau)$; one can obtain information on $T(\tau)$ from independent measurements or by instrumental or numerical integration applied to measurements on $V(\tau)$; (25) is simple and convenient. We supplement it with expressions for λ and c:

$$c = \frac{q}{V} \left(\frac{\pi r^2}{\eta'} \right)^d \exp(-\eta'), \quad (26)$$

$$\lambda = \frac{q}{V} \frac{\pi^d}{4\tau} \left(\frac{r^2}{\eta'} \right)^{d+1} \exp(-\eta'), \quad (27)$$

where $\eta' = \eta-(d+2)$.

We now consider instantaneous-heating methods; (9) and (4) coincide apart from a constant factor, so most of the results for V_c can be transferred in a formal fashion to T_δ.

We consider V/T and omit the subscript δ here and subsequently to get

$$a = r^2/4\tau(\eta - d), \quad (28)$$

where $\eta = V\tau/T = T'\tau/T$ is the $T(\tau)$ step parameter:

$$c = \frac{Q}{T} \left(\frac{\pi r^2}{\eta - d} \right)^d \exp(d - \eta); \quad (29)$$

$$\lambda = \frac{Q\pi^d}{4T\tau} \left(\frac{r^2}{\eta - d} \right)^{d+1} \exp(d - \eta). \quad (30)$$

We find the turning points in (5), for which we solve $(V)'_\tau = 0$:

$$\tau_{\max} = \sigma^2/\sqrt{1-d}\,(\sqrt{1-d}+1), \quad (31)$$

$$\tau_{\min} = \sigma^2/\sqrt{1-d}\,(\sqrt{1-d}-1). \quad (32)$$

We find the zeros in (5), $(\sigma^2/\tau_z) + d = 0$:

$$\tau_z = \sigma^2/(-d). \quad (33)$$

From (31)-(33) we have

$$a = r^2/4\tau_{\max}\sqrt{1-d}\,(\sqrt{1-d}+1), \tag{34}$$

$$a = r^2/4\tau_{\min}\sqrt{1-d}\,(\sqrt{1-d})-1), \tag{35}$$

$$a = r^2/4\tau_z(-d). \tag{36}$$

Deriving the turning points is poorly conditioned because the radius of curvature for $V(\tau)$ is insufficiently small near the maximum and (especially) the minimum. Then if one requires higher accuracy in determining a, it is better to use (36), since $V'(\tau_z)$ is sufficiently large. We use (36) and (5) written for any other τ^* for which $V'(\tau^*)$ is sufficiently large to define c and λ with the required accuracy:

$$c = \frac{Qd}{V(\tau^*)\tau^*}\left[\frac{\pi r^2\tau^*}{(-d)\tau_z}\right]^d\left(1-\frac{\tau_z}{\tau^*}\right)\exp\left(d\,\frac{\tau_z}{\tau^*}\right), \tag{37}$$

$$\lambda = \frac{Qd\,(\pi\tau^*)^d}{4V(\tau^*)\tau^*}\left[\frac{r^2}{(-d)\tau_z}\right]^{1+d}\left(1-\frac{\tau_z}{\tau^*}\right)\exp\left(d\,\frac{\tau_z}{\tau^*}\right). \tag{38}$$

When (34)-(36) are used, we are rigidly restricted to $\tau_{\max}$, $\tau_{\min}$, and τ_z: we therefore propose a formula for any τ. Logarithmic differentiation applied to (5) gives

$$\eta = \frac{V'\tau}{V} = (d-1)+\frac{(d-n)(1-n)}{n}, \quad n = \frac{\sigma^2}{\tau}+d,$$

so

$$a = r^2/4\tau[\varkappa - d - \operatorname{sign}(\tau-\tau_z)\sqrt{\varkappa^2-d}], \tag{39}$$

where $\varkappa = \eta/2+1$; $\operatorname{sign}(\tau-\tau_z) = \{1;\ \tau<\tau_z;\ 0;\ \tau=\tau_z;\ -1;\ \tau>\tau_z\}$, with τ_z the zero of (5).

This is an exact formula if $\eta = V'\tau/V$ is derived exactly; many of the previous expressions contain $\eta = V'\tau/V$ ($\eta = T'\tau/T$), which itself contains the derivative V', which is not measured directly, so V' can be derived by differentiating the measured $V(\tau)$. That is possible if the curve is first smoothed, but it is not always possible to get the required accuracy. However, all the expressions containing η are of undoubted value, since they enable one to solve the problem, which is one of separating roots on using iterative methods to solve algebraic equations. When one knows the range in which the exact solution lies, one can employ methods having guaranteed convergence such as division into halves [7], which can be combined with some method of accelerating convergence such as Aitken's method [8], which enables one to find the root with given accuracy quite rapidly.

If we transfer this to our case, (39) and similar expressions containing η represent the exact solution of $F(a) = 0$, e.g., as in (5).

We used the descending differences $V'_d \approx (V_1-V_2)/\Delta\tau$ and the ascending ones $V'_a \approx (V_2-V_{-1})/\Delta\tau$ [9] to get the left boundary

$$a_a = r^2/4\tau\left[\left(\frac{\eta_a}{2}+1\right)-d+\sqrt{\left(\frac{\eta_a}{2}+1\right)^2-d}\right] \tag{40}$$

and the right boundary

$$a_d = r^2/4\tau\left[\left(\frac{\eta_d}{2}+1\right)-d+\sqrt{\left(\frac{\eta_d}{2}+1\right)^2-d}\right] \tag{41}$$

of the root of

$$V(\tau_2)-a\tau_2^{d-1}\left[\frac{r^2}{4a\tau_2}+d\right]\exp\left(-\frac{r^2}{4a\tau_2}\right) = 0, \tag{42}$$

where

$$\eta_a = \left(1-\frac{V_{-1}}{V_2}\right)\frac{\tau_2}{\Delta\tau}; \quad \eta_d = \left(\frac{V_1}{V_2}-1\right)\frac{\tau_2}{\Delta\tau}.$$

The above method gives the root a_n with a given accuracy; frequently, the substitution $\eta_1 = (\eta_a+\eta_d)/2$ into (39) gives the root with sufficient accuracy.

If numerical values for d are substituted into (31)-(32), we get the following pattern:

$$\tau_{\max 1} \approx 0.25\sigma^2, \quad \tau_{\max 2} \approx 0.29\sigma^2, \quad \tau_{\max 3} \approx 0.37\sigma^2,$$

$$\tau_{z1} = \frac{2}{3}\sigma^2, \quad \tau_{z2} = \sigma^2, \quad \tau_{z3} = 2\sigma^2.$$

The $\tau_{\max}$ spread is slight, and $\tau_{z3} = 3\tau_{z1}$, i.e., the process for a planar source is three times slower than for a point one as regards attaining τ_z, so from the viewpoint of lag, a system based on a point source is better than a linear or planar one.

The transfer from measuring $T(r,\tau)$ to measuring $V(r,\tau)$ thus provides a series of simple and quite accurate formulas for λ, c, and a for stepped or instantaneous heat sources acting in unbounded bodies.

We see from (6)-(8) and (9) that there is a considerable simplification on going from temperature measurement to measurement of the rate of change.

In conclusion, we neglect measurement errors in estimating the relative errors of the approximations (14) and (25).

For $\sigma^2/\tau \sim 10$, i.e., at the start, where $\Delta\tau/\tau \sim 10^{-2}$, (14) has a relative error of about 3%; for $\sigma^2/\tau \sim 10$, (25) gives a relative error of about 0.5%, i.e., (14) and (25) can be used on the same basis as the accurate formulas. Calculations show that the time required to give a reliable measurement of the rate V_δ in nonstationary-frequency methods is less by an order of magnitude than the time required to give a reliable measurement of T_δ by amplitude methods, which confirms that the nonstationary-frequency methods are more sensitive and rapid.

NOTATION

T, temperature; V, rate of change of temperature; Q, amount of heat; q, heat flux; a, thermal diffusivity; c, specific heat; x, y, z, spatial coordinates; τ and t, time; $E_1(u) = -Ei(-u)$, integral exponential function; erfc(u) = 1 − erf(u), where erf(u) is the error function; $\operatorname{ierfc}(u) = \int_x^\infty \operatorname{erfc}\xi d\xi = \frac{1}{\sqrt{\pi}}\exp(-u^2) - u \operatorname{erfc} u$.

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